Assignment Problem Problems[1]

Math 20

Due: October 15, 2004

1. Find an optimal assignment and corresponding cost for each of the following three cost matrices.

   a. $\begin{bmatrix} 17 & 4 & 10 \\ 15 & 5 & 8 \\ 18 & 7 & 11 \end{bmatrix}$

   b. $\begin{bmatrix} 3 & -2 & 0 & 1 \\ 5 & 3 & -3 & 4 \\ 2 & 7 & 5 & 3 \\ 5 & -2 & 0 & 1 \end{bmatrix}$

   c. $\begin{bmatrix} 12 & 9 & 7 & 7 & 10 \\ 15 & 11 & 8 & 13 & 14 \\ 9 & 6 & 5 & 12 & 12 \\ 6 & 9 & 13 & 7 & 10 \\ 8 & 13 & 12 & 9 & 13 \end{bmatrix}$

2. A coin dealer is to sell four coins through a mail auction. Bids are received for each of the four coins from five bidders with instructions that at most one of his bids is to be honored. The bids are:

	Bids			
	Coin 1	Coin 2	Coin 3	Coin 4
Bidder 1	$150	$65	$210	$135
Bidder 2	175	75	230	155
Bidder 3	135	85	200	140
Bidder 4	140	70	190	130
Bidder 5	170	50	200	160

   a. How should the dealer assign the four coins in order to maximize the sum of the resulting bids? (Notice that a "dummy" coin must be added to produce a square cost matrix. Whichever bidder receives the dummy coin does not receive any real coin.) Also keep in mind that we don't want to *minimize* the sum of the resulting bids!
   b. Suppose that bidder 2 instructed that dealer that at most *two* of his bids are to be honored. How can the problem be modified, and what is an optimal assignment for the new problem?

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[1] See Section 11.4 of *Elementary Linear Algebra, Applications Version* by Howard Anton and Chris Rorres.

3. Prove this step in the Hungarian algorithm: If an assignment of zeros exists, the zeros in the matrix cannot be covered with fewer than n lines.
4.
   a. Let the zero entries of an n by n nonnegative cost matrix C be covered with m lines, with $m \leq n$, Let $a \geq 0$ be the smallest uncovered entry, and let C' be the cost matrix obtained by applying Step 5 of the Hungarian method to C. Show that

   $$(\text{sum of all entries of } C) - (\text{sum of all entries of } C') = an(n - m)$$

   b. Use the result of part (a) to show that the Hungarian method will produce a cost matrix with an optimal assignment of zeros in a finite number of steps.